Customizing Modern Portfolio Theory for the Project Portfolio Selection Problem

ABSTRACT

As organization's performance depends on the projects it implements, selecting the most appropriate set of projects given limited resources is a crucial decision. In such project portfolio selection decision, the combined analysis of portfolios' returns and risks, i.e. risk-return optimization, is essential since a project portfolio with a high attractive expected return might also expose the organization to a large loss. Furthermore, as these two variables are influenced by some external threats and opportunities that may affect the returns of one or more projects simultaneously, it is crucial to incorporate such effects into the optimization model. However, the literature is underdeveloped in such critical incorporation. Inspired by "modern portfolio theory", an effective approach in (financial) portfolio selection problem, in this paper, we propose a new approach to solve the project portfolio selection problem, which comprehensively considers the effects of threats and opportunities around projects in the risk-return optimization model. To demonstrate how to apply the proposed new approach, we employ a numerical example and report the results.

Keywords:

Project portfolio selection problem; risk-return optimization; modern portfolio theory; risk register; correlation

1. INTRODUCTION

Organizations approve project proposals that can help them achieve their strategic objectives. However, because resources are limited, selecting the most appropriate set of projects is a strategic organizational decision (Ghasemzadeh, Archer, & Iyogun, 1999). Accordingly, project portfolio selection problem (PPSP) determines the subset of projects to be funded to optimize organizational performance objective without violating indispensable constraints (Li, Fang, Tian, & Guo, 2015; Lorie & Savage, 1955; Shou, Xiang, Li, & Yao, 2014). Whereas the portfolio's return is considered by the literature as the most important corporate goal to be maximized, an effective organization's portfolio should also be examined by its risk. This is because a portfolio with a very attractive expected return might expose the organization to a large loss, whereas a low-risk portfolio might secure the organization lower but more certain return. Thus, the combined analysis of risk and return ("risk-return optimization") is important in this context (Sefair, Méndez, Babat, Medaglia, & Zuluaga, 2016).

This kind of optimization is well-developed in (financial) portfolio selection problem (PSP), which is one of the most studied topics in finance and is concerned with the allocation of limited capital to a number of potential assets for a profitable investment strategy (Lwin & Qu, 2013). An effective approach to risk-return optimization in PSP is “Modern portfolio theory” (MPT)-for which its pioneer, Harry Markowitz, was awarded a Nobel Prize (Varian, 1993). MPT views PSP as a mean-variance optimization problem with regard to two criteria: to maximize the portfolio’s return and to minimize its risk, measured by the mean and variance of its return respectively (Markowitz, 1952).

The underlying principle of MPT is that assets should not be selected solely based on their individual merits (i.e., expected returns), so it is concerned with quantifying how each asset' return changes in relation to other assets in the portfolio and overall market fluctuations (Casault, Groen, & Linton, 2013). This crucial principle also applies to the projects in a portfolio as their expected returns are affected by external factors, in particular the threats and opportunities, which in turn can result in correlations among projects. For example, a change in the inflation rate can increase the expected return of an asset or a project, as an opportunity, and simultaneously reduce that of another asset or project, as a threat.

Having considered the similar concepts between PSP and PPSP such as return, risk, correlation and the available budget, researchers tried to apply MPT into the risk-return optimization of PPSP (e.g., Esfahani, Sobhiyah, & Yousefi, 2016; Luo, 2012; Sefair et al., 2016). However, there is a common limitation in their studies as they disregarded the fundamental differences that exist between the characteristics of financial portfolios and those of project portfolios (Casault et al., 2013). For example, unlike financial assets, often there is not enough projects' historical data to calculate different MPT's parameters or the accessible historical data is not reliable enough, as the threats and opportunities faced by a project can be unique and not possible to generalize to other projects. In order to address this limitation, the present paper first distinguishes the essential differences between PSP and PPSP and then proposes some customizations in the methods applied to estimate MPT's parameters in order to make it suitable to be used in PPSP. It is expected that the proposed approach opens a new horizon in the structure of solutions for PPSP and improves the reliability of such decision making in organizations by answering the following research question: “How to incorporate the effects of different threats and opportunities around projects into risk-return optimization in PPSP?”

The rest of the paper is organized as follows. Section 2 reviews different approaches have been used by the researchers into PPSP, describes MPT in brief and exemplifies some studies have applied it in PPSP. The formulation of the problem which customize MPT to PPSP is proposed in Section 3. Section 4 presented a numerical example to demonstrate how the proposed model should be used. Finally, the conclusions are drawn in Section 5.

2. LITERATURE REVIEW

2.1 Different approaches to PPSP

PPSP arises from the everyday dilemma faced by organizations in finding the best possible way to distribute a limited budget among project proposals to fulfil their strategic objectives (Carazo, 2015). Making a wrong decision can result in two destructive consequences: (1) resources are wasted on inappropriate projects that have been funded (type II error), and (2) the benefits that could have been realized from allocating such resources to better projects are lost (type I error) (Christensen & Knudsen, 2010; Martino, 1995). Thus, researchers have developed various approaches to solve PPSP since the mid-1950s (Lorie & Savage, 1955) that can be categorized into four distinct groups.

The first group of approaches consider only one criterion to assign a score to each project. Thus, projects are selected from the highest to the lowest score of this criterion until the budget available is spent. For instance, Lorie and Savage (1955), Myers (1972) and Weingartner (1962) applied economic assessment measurements (e.g., net present value and internal rate of return) to calculate the project score.

As the sphere in which decisions are taken in any organization is usually characterized by a set of competing criteria (Carazo, 2015), the second category of methods present different multi-criteria approaches in order to incorporate the decision maker's preferences into the process. For example, Gear, Lockett and Muhlemann (1982), Melachrinoudis and Rice (1991) as well as Versapalainen and Lauro, (1988) applied "comparative models" such as analytical hierarchy process (AHP) to combine different criteria to a single objective criterion and then compare one project to either another project or some subsets of alterative projects. Albala (1975), Cooper (1978) and Krawiec (1984) used "Scoring approaches" to combine the merit of each project with respect to a small number of decision criteria to specify its desirability score and then rank projects according to their scores. Benjamin (1985), Golabi, Kirkwood and Sicherman (1981) as well as Neely, North and Fortson (1977) applied "mathematical programing models" to maximize objectives such as profit, revenue and utility or minimize others like resource use, cost and runtime simultaneously.

As there may be technical (i.e. complementarities and incompatibilities), resource and benefit interactions (i.e. synergies produced by sharing costs and benefits respectively) among projects derived from conducting more than one project at the same time (Czajkowski & Jones, 1986; Fox, Baker, & Bryant, 1984), the third group of solutions incorporate one or more categories of these interactions into the decision model. For example, Carraway and Schmidt (1991) proposed a model by formulizing the benefit and resource interactions among pairs of projects quantitatively. Klapka and Piños (2002), Lee and Kim (2000), Santhanam and Kyparisis

(1995) as well as Schmidt (1993) suggested different methodologies which reflect benefit, resource and technical interactions among the sets of two or three projects. Doerner, Gutjahr, Hartl, Strauss and Stummer (2006), Santhanam and Kyparisis (1996) as well as Yu, Wang, Wen and Lai (2012) developed various models that allowed the technical, benefit and resource interactions among any numbers of projects.

A major limitation of all above-mentioned approaches was disregarding the fact that “no project is an island” (Engwall, 2003) and so they have relationships with the outer context (Rungi, 2010). In other words, it is crucial to reflect the effects of external factors on projects’ returns, which in turn result in projects risks and correlation among projects, in solutions used for PPSP (Chiu & Gear, 1979; Gear & Cowie, 1980). This argument resulted in the advent of the fourth group of approaches which try to consider projects’ returns and risks as well as correlations among projects in their decision models by employing MPT which is an effective approach for this type of analysis, i.e. risk-return optimization, in PSP. In the following of this section we first take a look at the structure of MPT and then summarise some approaches used it in PPSP.

2.2 Modern Portfolio Theory (MPT)

One of modern finance theory’s main tenets is MPT which led to Markowitz’s Noble Prize in Economics in 1990. Markowitz (1952) originally formulated the fundamental theorem of mean-variance portfolio framework for risk-return optimization in PSP, which trades off between expected return and risk of a portfolio (represented by mean and variance of that portfolio’s return respectively) to reach the optimal portfolio of various assets. A portfolio is also considered to be efficient if for a given level of risk there are no portfolios with a higher expected return, or conversely for a given expected return there are no portfolios with a lower risk. The complete set of these efficient portfolios forms the efficient frontier that represents the best trade-offs between return and risk (Markowitz, 1952, 1959; Markowitz, Todd, & Sharpe, 2000). Accordingly, the formulation of the mean-variance optimization is as follows (Bodie, Kane, & Marcus, 2014):

$$Max\, S_p = \frac{E(r_p) - r_f}{\sigma_p} \tag{1}$$

$$Where\ E(r_p) = \sum_{i=1}^{N} X_i \mu_i \tag{2}$$

$$\sigma_p^2 = \sum_{i=1}^{N} \sum_{j=1}^{N} X_i X_j \sigma_i \sigma_j \rho_{ij} \tag{3}$$

$$Subject\, to \sum_{i=1}^{N} X_i = 1 \tag{4}$$

$$0 \le X_i \le 1 \qquad i = 1, 2, ..., N \tag{5}$$

Where,

S_p is the slope of capital allocation line (*CAL*), called “Sharpe Ratio” (Sharpe, 1994),

$E(r_p)$ is the expected return of portfolio *p*,

r_f is the risk-free return (e.g., the return of placing money in the bank),

σ_p is the standard deviation of return relevant to portfolio *p,*

μ_i is the expected return of asset *i,*

σ_i is the standard deviation of return relevant to asset *i,*

ρ_{ij} is "Pearson correlation coefficient" between assets *i* and *j,*

N is the number of available assets, and

X_i is the decision variable of the budget proportion invested in asset *i.*

To estimate two parameters μ_i and σ_i, MPT uses the historical data relevant to asset i as follows:

$$\mu_i = \sum_{s_i=1}^{M_i} p(s_i) r(s_i) \tag{6}$$

$$\sigma_i^2 = \sum_{s_i=1}^{M_i} p(s_i) \left(r(s_i) - \mu_i \right)^2 \tag{7}$$

Where,

s_i is the s^{th} historical situation relevant to asset *i*,

M_i is the number of historical situations available for asset *i*,

$r(s_i)$ is the return of asset *i* in historical situation s_i, and

$p(s_i)$ is the proportion of happening historical situation s_i.

Figure 1 depicts the "optimal capital allocation line", *CAL (P)*, which is tangent to the "efficient frontier" at the "optimal portfolio" (*P*).

Insert Figure 1 about here

Having summarized the approach of MPT, that is both well-constructed in its theoretical foundations and successful in its applications in PSP (Chien, 2002), we can now turn to the fourth group of approaches in PPSP.

2.3. Applying MPT in PPSP

Researchers have identified important similarities between PSP and PPSP (e.g., Boasson, Cheng, & Boasson, 2012; Luo, 2012; Sefair et al., 2016). Table 1 summarizes these similarities distinguished in four concepts, i.e. return, risk, correlation and available budget, which justifies using MPT in PPSP.

Insert Table 1 about here

Accordingly, the fourth group of studies apply the mean-variance foundation of MPT in forming optimal risk-return portfolio in PPSP. For instance, Boasson et al. (2012) applied MPT to municipal financial and capital budgeting decisions by considering the historical data relevant to benefits and cost of the similar projects. Esfahani et al. (2016) applied MPT to PPSP considering the historical returns of similar projects and proposed the harmony search algorithm to solve it. Luo (2012) concentrated on the risk-side control for Research and Development (R&D) project portfolio and developed a method for optimal diversification of R&D project portfolio incorporating market and technology risk by using projects' historical data. Morcos (2008) introduced a methodology for R&D projects that integrated financial returns of these projects and their reliability into a total benefit dimension that was traded-off against their costs. Sefair et al. (2016) developed a linear solution for the Mean-SemiVariance PPSP in the oil and gas industry by considering the historical data relevant to the net present value of the similar projects.

An analysis of this group of approaches applying MPT in PPSP suggests that they ignore some fundamental differences that exist between PSP and PPSP. In particular, as the threats and opportunities that surround a project can be unique, it is not logical to use projects' historical data to estimate MPT's parameters, i.e. projects' returns and risks as well as the correlation coefficients among them, in PPSP. In other words, these studies are underdeveloped in incorporating the effects of such threats and opportunities into the risk-return optimization of PPSP. In the following, we customize MPT in PPSP by first illuminating all differences exist between PSP and PPSP, and then developing suitable approaches to address these differences.

3. PROBLEM FORMULATION

3.1. Fundamental differences between PSP and PPSP

Some researchers have highlighted differences between financial portfolios and project portfolios. Bakhshi and Touran (2012) asserted that usually there is not sufficient historical data available to calculate the correlation coefficients among projects. Boasson et al. (2012) and Casault et al. (2013) argued that unlike financial assets which only have monetary benefits, real projects have both monetary and non-monetary benefits.

Table 2 compares the approaches used in PSP with those should be used in PPSP which introduces some initial guidelines for the proposed model.

Insert Table 2 about here

According to substantial differences between PSP and PPSP highlighted in Table 2, in particular the lack of reliable projects' historical data rooted in different threats and opportunities exist around projects, it can be concluded that despite the similar concepts between two problems mentioned in Table 1, direct application of MPT to PPSP is problematic and some customizations in the estimation methods of MPT's parameters are needed to make it suitable to be used in PPSP. In the rest of this section we develop the mathematical model by proposing novel approaches to address such customizations.

3.2. Customizing MPT to PPSP

A project life includes four phases: initiation, planning, execution and outcome realization (Zwikael & Smyrk, 2011). The decision whether a project gets funded is made at the first phase of projects' life, i.e. initiation. One of the main project' documents prepared in this phase is "risk register" (Project Management Institute, 2013). Risk register is the primary tool in risk management, which is used to document the results of analysis and outline the mitigation program being proposed for each risk (Zwikael & Smyrk, 2011). As this document is a comprehensive document of all threats and opportunities exist around a projects, this paper uses some data mentioned in this document to estimate MPT's parameters in order to customize MPT for PPSP. This approach would be helpful and practical, as it removes any other additional required efforts. As an example, by using this approach there is no need to search about the projects' historical data that are not finally reliable enough as the characteristics of each project are unique. In the following, we first introduce our model's assumptions, second define the appropriate format for risk register, then propose a customized approach to estimate MPT's parameters in PPSP and finally develop the mathematical model.

3.2.1. Model assumptions

To develop the mathematical model, we consider some assumptions as follows:

- We consider the characteristics of projects before doing any risk mitigations. This assumption provides a fair and logical situation for different projects' proposals to compare.
- We assume that an organization is going to select its most appropriate project portfolio to implement in a special period of time. In other words, we are not consider the scheduling of projects in the portfolio.
- We hypothesize that projects are not divisible, that is to say, the decision variables are binary, representing the selection, or not, of each project proposal.

3.2.2. The Risk register

There are different formats proposed by various studies and standards for developing a risk register. One of the most comprehensive formats is that proposed by Zwikael and Smyrk (2011), in which a risk register is a table where rows are associated with threats and columns are relevant to their attributes. Having considered the first model assumption, we use the first four columns of their proposed format as follows:

1) Threat ID
2) Threat title: description of the triggering event

3) Pre-likelihood of the threat in the absence of the proposed mitigation action
4) The damaging impacts of the threat on the project' return as at least one of the six impacts: benefit reduced, benefit delayed, disbenefit increased, disbenefit advanced, cost increased and cost advanced

Furthermore, in order to generalize our formulation, we apply some modifications in the above-mentioned risk register's format as follows:

- We consider both threats and opportunities in the first and second columns to cover the external factors comprehensively. Thus, the third column demonstrates the pre-likelihood of happening corresponding threat or opportunity. Furthermore, there are six possible contributory impacts of the opportunities, which are benefit increased, benefit advanced, disbenefit decreased, disbenefit delayed, cost decreased and cost delayed, in addition to the above-mentioned potential damaging impacts of threats in the fourth column.
- We consider the present value of all projects' benefits, disbenefits and costs to provide a baseline time as a fair condition for all project proposals to compare.

Accordingly, Table 3 shows the required part of a risk register that is used in estimating MPT's parameters explained in the rest of this section.

Insert Table 3 about here

3.2.3. The customized approach to estimate MPT's parameters (projects' returns and risks as well as correlation coefficients among projects) in PPSP:

In the course of a project's life, any number of the threats and opportunities mentioned in its risk register can happen. Thus, the total number of potential situations can surround project *i,* M_i, is 2^{n_i} reached from Equation (8) in which n_i is the number of threats and opportunities mentioned in the risk register of project *i*.

$$M_i = \binom{n_i}{0} + \binom{n_i}{1} + \binom{n_i}{2} + \binom{n_i}{3} + \ldots + \binom{n_i}{n_i} = 2^{n_i} \tag{8}$$

On the other hand, we define projects' returns as the ratio of benefits minus disbenefits minus costs to costs, in which benefits and disbenefits can be both monetary and non-monetary. Furthermore, according to Table 3, each threat and opportunity can affect the "magnitude" or "scheduling for realization" of different benefits, disbenefits or costs. Thus, the return of project *i* in potential situation s_i, $r(s_i)$, is calculated as follows:

$$r(s_i) = \frac{B(s_i) - D(s_i) - C(s_i)}{C(s_i)} \tag{9}$$

$$Where\ B(s_i) = \sum_{k=1}^{K_i} M_{B_k} \left(1 + \sum_{r \in s_i} \Delta M_{B_{kr}}\right) e^{-\alpha S_{B_k}\left(1+\sum_{r \in s_i} \Delta S_{B_{kr}}\right)} \tag{10}$$

$$D(s_i) = \sum_{l=1}^{L_i} M_{D_l} \left(1 + \sum_{r \in s_i} \Delta M_{D_{lr}}\right) e^{-\alpha S_{D_l}\left(1+\sum_{r \in s_i} \Delta S_{D_{lr}}\right)} \tag{11}$$

$$C(s_i) = \sum_{f=1}^{F_i} M_{C_f} \left(1 + \sum_{r \in s_i} \Delta M_{C_{fr}}\right) e^{-\alpha S_{C_f}\left(1+\sum_{r \in s_i} \Delta S_{C_{fr}}\right)} \tag{12}$$

Where,

s_i is the s^{th} situation in project *i* out of 2^{n_i} potential situations,

$B(s_i)$ is the total present value of benefits relevant to project *i* if potential situation s_i happens,

$D(s_i)$ is the total present value of disbenefits relevant to project *i* if potential situation s_i happens,

$C(s_i)$ is the total present value of costs relevant to project *i* if potential situation s_i happens,

K_i is the total number of benefits relevant to project *i*,

L_i is the total number of disbenefits relevant to project *i*,

F_i is the total number of costs relevant to project *i*,

M_{B_k} is the estimated magnitude of k^{th} benefit relevant to project *i*,

$\Delta M_{B_{kr}}$ is the estimated percentage of changes in the magnitude of k^{th} benefit relevant to project *i* if threat/opportunity *r* happens,

S_{B_k} is the estimated scheduling for the realization of k^{th} benefit relevant to project *i*,

$\Delta S_{B_{kr}}$ is the estimated percentage of changes in the scheduling for the realization of k^{th} benefit relevant to project *i* if threat/opportunity *r* happens,

M_{D_l} is the estimated magnitude of l^{th} disbenefit relevant to project *i*,

$\Delta M_{D_{lr}}$ is the estimated percentage of changes in the magnitude of l^{th} disbenefit relevant to project *i* if threat/opportunity *r* happens,

S_{D_l} is the estimated scheduling for the realization of l^{th} disbenefit relevant to project *i*,

$\Delta S_{D_{lr}}$ is the estimated percentage of changes in the scheduling for the realization of l^{th} disbenefit relevant to project *i* if threat/opportunity *r* happens,

M_{C_f} is the estimated magnitude of f^{th} cost relevant to project *i*,

$\Delta M_{C_{fr}}$ is the estimated percentage of changes in the magnitude of f^{th} cost relevant to project i if threat/opportunity r happens,

S_{C_f} is the estimated scheduling for the realization of f^{th} cost relevant to project i,

$\Delta S_{C_{fr}}$ is the estimated percentage of changes in the scheduling for the realization of f^{th} cost relevant to project i if threat/opportunity r happens,

α is discount rate, and

e is Euler's number.

On the other hand, the probability of happening situation s_i, $p(s_i)$, is calculated as the multiplication of the pre-likelihoods of happening the threats/opportunities included in situation s_i and pre-likelihoods of not happening the others, demonstrated as follows:

$$p(s_i) = \prod_{r \in s_i} l_r \prod_{r \notin s_i} (1 - l_r) \tag{13}$$

Where l_r is the pre-likelihood of happening threat/opportunity r.

After placing Equations (8), (9) and (13) in Equations (6) and (7), the final customized formulations for return (μ_i) and risk (standard deviation, σ_i) relevant to project i can be reached as follows:

$$\mu_i = \sum_{s_i=1}^{2^{n_i}} \left(\prod_{r \in s_i} l_r \prod_{r \notin s_i} (1 - l_r) \right) \left(\frac{B(s_i) - D(s_i) - C(s_i)}{C(s_i)} \right) \tag{14}$$

$$\sigma_i^2 = \sum_{s_i=1}^{2^{n_i}} \left(\prod_{r \in s_i} l_r \prod_{r \notin s_i} (1 - l_r) \right) \left(\frac{B(s_i) - D(s_i) - C(s_i)}{C(s_i)} - \mu_i \right)^2 \tag{15}$$

Where $B(s_i)$, $D(s_i)$ and $C(s_i)$ are calculated by Equations (10), (11) and (12) respectively.

Having calculated returns and risks of projects, in the following, we propose a methodology to calculate correlation coefficients among projects in a portfolio. Each one of the projects in the pairs of projects has two categories of threats/opportunities: particular threats/opportunities (which are specific to each one) and common ones (which are common in the two projects). It should also be mentioned that a threat for a project can be a threat or opportunity to another project. To reach the correlation coefficient between projects *i* and *j*, ρ_{ij}, we first calculate their covariance, COV_{ij}, as follows:

$$
\begin{aligned}
COV_{ij} &= COV(\mu_i , \mu_j) \\
&= COV\left(\sum_{s_i=1}^{2^{n_i}} p(s_i)r(s_i), \sum_{s_j=1}^{2^{n_j}} p(s_j)r(s_j) \right) \\
&= COV\left(\sum_{c_{ij}=1}^{2^{n_{ij}}} p(c_{ij})r(c_{ij}) + \sum_{p_i=1}^{2^{n_i}-2^{n_{ij}}} p(p_i)r(p_i), \sum_{c_{ji}=1}^{2^{n_{ji}}} p(c_{ji})r(c_{ji}) + \sum_{p_j=1}^{2^{n_j}-2^{n_{ji}}} p(p_j)r(p_j) \right) \\
&= COV\left(\sum_{c_{ij}=1}^{2^{n_{ij}}} p(c_{ij})r(c_{ij}), \sum_{c_{ji}=1}^{2^{n_{ji}}} p(c_{ji})r(c_{ji}) \right) + \\
&\quad COV\left(\sum_{c_{ij}=1}^{2^{n_{ij}}} p(c_{ij})r(c_{ij}), \sum_{p_j=1}^{2^{n_j}-2^{n_{ji}}} p(p_j)r(p_j) \right) + \\
&\quad COV\left(\sum_{p_i=1}^{2^{n_i}-2^{n_{ij}}} p(p_i)r(p_i), \sum_{c_{ji}=1}^{2^{n_{ji}}} p(c_{ji})r(c_{ji}) \right) + \\
&\quad COV\left(\sum_{p_i=1}^{2^{n_i}-2^{n_{ij}}} p(p_i)r(p_i), \sum_{p_j=1}^{2^{n_j}-2^{n_{ji}}} p(p_j)r(p_j) \right)
\end{aligned}
\tag{16}
$$

Where,

n_{ij} is equal to n_{ji} and is the number of common threats and opportunities in projects *i* and *j*,

c_{ij} and c_{ji} are the c^{th} common situations in projects *i* and *j* respectively,

$p(c_{ij})$ and $p(c_{ji})$ are the probabilities of happening common situations c_{ij} and c_{ji} respectively,

$r(c_{ij})$ and $r(c_{ji})$ are the returns of projects *i* and *j* in common situations c_{ij} and c_{ji} respectively,

p_i is the p^{th} particular situation in project *i*,

$p(p_i)$ is the probability of happening particular situation p_i in project *i*, and

$r(p_i)$ is the return of project *i* in particular situation p_i.

As we can see in equation (16), there are four terms which should be calculated to reach the total covariance between projects *i* and *j*. By considering some assumptions as there are no correlations among particular threats/opportunities of project *i* and those of project *j*, and there are no correlations among common threats/opportunities in project *i* and particular ones in project *j* and vice versa, the second, third and fourth terms of Equation (16) are equal to zero. These assumptions of independence are justified because no explicit relationships exist among these combinations of threats/opportunities. In other words, if one occurs in project *i*, it does not give us any new information on occurrence of the other one in Project *j* (Bakhshi & Touran, 2012). for the same reason, in the first term of Equation (16), only the probabilities of happening common threats and opportunities in projects *i* and *j* should be considered in calculating $p(c_{ij})$ and $p(c_{ji})$, which makes these two probabilities equal to each other. On the other hand, it is clear that if there are no common threats/opportunities in projects *i* and *j*, their covariance would be equal to zero. Accordingly, the final formulation of covariance between projects *i* and *j* is as follows:

$$COV_{ij} = COV\left(\sum_{c_{ij}=1}^{2^{n_{ij}}} p(c_{ij}) r(c_{ij}), \sum_{c_{ji}=1}^{2^{n_{ji}}} p(c_{ji}) r(c_{ji}) \right)$$
$$= \begin{cases} 0 \quad \textit{If there are no common threats / opportunities between projects i and j} \\ \sum_{c_{ij}=1}^{2^{n_{ij}}} p(c_{ij}) \left(r(c_{ij}) - \sum_{c_{ij}=1}^{2^{n_{ij}}} p(c_{ij}) r(c_{ij}) \right) \left(r(c_{ji}) - \sum_{c_{ji}=1}^{2^{n_{ji}}} p(c_{ji}) r(c_{ji}) \right) \quad \textit{Otherwise} \end{cases} \tag{17}$$

Having considered the relationship between correlation coefficient and covariance as demonstrated in Equation (18), the final formulation of correlation coefficient between projects *i* and *j* is drawn from Equation (19).

$$\rho_{ij} = \frac{COV_{ij}}{\sigma_i \sigma_j} \tag{18}$$

$$\rho_{ij} = \begin{cases} 1 \quad \textit{If } i = j \\ 0 \quad \textit{If } i \neq j \ \& \\ \quad \textit{there are no common threats / opportunities between projects i and j} \\ \dfrac{\sum_{c_{ij}=1}^{2^{n_{ij}}} p(c_{ij}) \left(r(c_{ij}) - \sum_{c_{ij}=1}^{2^{n_{ij}}} p(c_{ij}) r(c_{ij}) \right) \left(r(c_{ji}) - \sum_{c_{ji}=1}^{2^{n_{ji}}} p(c_{ji}) r(c_{ji}) \right)}{\sigma_i \sigma_j} \quad \textit{Otherwise} \end{cases} \tag{19}$$

3.2.4. Mathematical model

Having considered all materials mentioned so far, the final mathematical model is reached as follows:

$$Max\, S_p = \frac{E(r_p)}{\sigma_p} \tag{20}$$

$$Where\;\; E(r_p) = \sum_{i=1}^{N} X_i w_i \mu_i \tag{21}$$

$$\sigma_p^2 = \sum_{i=1}^{N} \sum_{j=1}^{N} X_i X_j w_i w_j \sigma_i \sigma_j \rho_{ij} \tag{22}$$

$$Subject\, to \sum_{i=1}^{N} B_i X_i \leq B \tag{23}$$

$$X_i = \{0,1\} \quad i = 1,2,...,N \tag{24}$$

Where,

μ_i is the expected return of project *i*, which is calculated by Equation (14),

σ_i is the risk (standard deviation) of project *i*, which is calculated by Equation (15),

ρ_{ij} is the correlation coefficient between projects *i* and *j*, which is drawn from Equations (19),

B is the total amount of available budget,

B_i is the estimated required budget to implement project *i*,

w_i is the proportion of total available budget required to invest in project *i* and is equal to the ratio of B_i to B ,

N is the number of project proposals, and

X_i is a binary decision variable, representing the selection, or not, of project *i.*

It should also be mentioned that as we consider the present value of expected return in our model, we ignore the risk-free return (r_f) in the objective function. In the next section, we employ a numerical example to show how the above-mentioned model should be used.

4. A NUMERICAL EXAMPLE

The purpose of this section is to employ a numerical example in order to illustrate how the proposed customized MPT can be used in PPSP. Let us assume that an organization considers 20 project proposals. However, as the organization has a limited budget of $300,000, it should select the best set of projects to execute. The organization needs to develop risk registers corresponding to each project proposal. Tables 4 and 5 depict two required parts of the risk registers developed for projects 1 and 2 respectively. As can be seen, there are two common external factors in projects 1 and 2 as first, P1.T01/P2.T01 ("Inflation rate increases") with the pre-likelihood of 0.3 and second, P1.T02/P2.O03 ("Iron import law is passed by the government") with the pre-likelihood of 0.2. Furthermore, it is derived that the former plays the role of threat for both projects, while the latter have a role of threat for one and that of opportunity for the other.

Insert Tables 4, 5 about here

As an example, we demonstrate how to calculate the return and risk of project 1. Regarding Equation (8), there are 8 potential situations in project 1. Table 6 shows these potential situations, the probability of happening each situation ($p(s_1)$) drawn from Equation (13), and the return of project 1 in each situation ($r(s_1)$) calculated by Equations (9) to (12).

Insert Table 6 about here

Having assumed discount rate as 0.09, the return and risk of project 1 are calculated as follows according to Equations (14) and (15) respectively:

$$\begin{aligned}\mu_1 = {} & (0.216\times 0.4185)+(0.126\times 0.5067)+(0.056\times 0.5562)+(0.054\times 0.3496)+ \\ & (0.024\times 0.3949)+(0.014\times 0.4815)+(0.006\times 0.3260)+(0.504\times 0.5813)=0.5154\end{aligned}$$

$$\begin{aligned}\sigma_1^2 = {} & 0.216\times(0.4185-0.5154)^2+0.126\times(0.5067-0.5154)^2+0.056\times(0.5562-0.5154)^2+ \\ & 0.054\times(0.3496-0.5154)^2+0.024\times(0.3949-0.5154)^2+0.014\times(0.4815-0.5154)^2+ \\ & 0.006\times(0.3260-0.5154)^2+0.504\times(0.5813-0.5154)^2=0.0064 \quad\Rightarrow\quad \sigma_1=0.0799\end{aligned}$$

Similarly, the return and risk of project 2 are reaches as 0.5055 and 0.0658 respectively.

To extract the correlation coefficient between projects 1 and 2, we consider their above-mentioned common threats/opportunities. According to Equation (8) there are four common situations in these two projects. Table 7 shows these common situations, the probability of happening each situation ($p(c_{12})$) drawn from Equation (13), and the returns relevant to projects 1 and 2 in each common situation (as represented by $r(c_{12})$ and $r(c_{21})$ respectively) have been calculated in the previous step.

Insert Table 7 about here

According to Equations (17), (18) and (19) the covariance and correlation coefficient between projects 1 and 2 are calculated as follows:

$$
\begin{aligned}
COV_{12} = COV_{21} = {} & 0.24 \times (0.4185 - 0.5179)(0.4789 - 0.5006) + \\
& 0.14 \times (0.5067 - 0.5179)(0.5287 - 0.5006) + \\
& 0.06 \times (0.3496 - 0.5179)(0.5053 - 0.5006) + \\
& 0.56 \times (0.5813 - 0.5179)(0.5023 - 0.5006) = 0.0005
\end{aligned}
$$

$$
\rho_{12} = \rho_{21} = \frac{0.0005}{0.0799 \times 0.0658} = 0.09
$$

Having exemplified how the proposed approach is applied to estimate returns and risks of project proposals as well as correlation coefficients among them, we now extract the optimal project portfolio. Table 8 shows the returns and risks relevant to all 20 project proposals.

Insert Table 8 about here

Furthermore, the correlation coefficient matrix relevant to all 20 projects are as follows:

$$
\rho = \begin{bmatrix}
1 & 0.09 & 0 & 0 & -0.02 & 0 & 0.03 & 0 & 0 & 0.08 & 0 & 0 & -0.23 & 0 & 0 & 0 & 0.12 & -0.21 & 0.32 & 0 \\
0.09 & 1 & 0 & 0.1 & 0 & 0 & -0.21 & 0 & -0.41 & 0.07 & 0 & -0.27 & 0 & 0.1 & 0 & -0.08 & 0 & 0.21 & 0 & 0.06 \\
0 & 0 & 1 & 0 & 0 & -0.07 & 0 & 0.22 & 0 & 0.09 & 0 & 0 & -0.12 & 0 & 0.03 & 0.01 & 0 & 0.11 & 0 & 0.23 \\
0 & 0.1 & 0 & 1 & 0 & 0.05 & 0 & 0.12 & 0 & -0.19 & 0 & 0 & 0 & 0.26 & 0 & 0.02 & 0.09 & -0.05 & 0 & 0 \\
-0.02 & 0 & 0 & 0 & 1 & 0 & 0.03 & 0.13 & 0.21 & 0 & -0.14 & 0 & -0.27 & 0 & 0 & -0.07 & 0 & 0 & 0.05 & 0.19 \\
0 & 0 & -0.07 & 0.05 & 0 & 1 & 0 & 0.19 & -0.11 & 0.09 & 0.21 & 0.06 & 0.03 & 0.18 & 0 & 0 & 0 & 0.08 & 0.09 & 0 \\
0.03 & -0.21 & 0 & 0 & 0.03 & 0 & 1 & 0 & 0.13 & 0 & 0 & 0.12 & 0 & -0.22 & 0.41 & 0 & 0.05 & 0.12 & -0.12 & 0.12 \\
0 & 0 & 0.22 & 0.12 & 0.13 & 0.19 & 0 & 1 & 0 & -0.19 & 0.08 & -0.20 & 0 & 0 & -0.11 & -0.08 & 0 & 0 & 0 & 0 \\
0 & -0.41 & 0 & 0 & 0.21 & -0.11 & 0.13 & 0 & 1 & 0 & 0.10 & 0 & 0.09 & 0.10 & 0.08 & 0.12 & -0.08 & 0 & 0 & -0.09 \\
0.08 & 0.07 & 0.09 & -0.19 & 0 & 0.09 & 0 & -0.19 & 0 & 1 & 0 & 0.05 & -0.05 & 0.12 & -0.14 & 0.32 & -0.06 & 0 & -0.19 & 0 \\
0 & 0 & 0 & 0 & -0.14 & 0.21 & 0 & 0.08 & 0.10 & 0 & 1 & 0 & -0.11 & 0 & 0.13 & 0 & 0 & 0 & 0.17 & 0.11 \\
0 & -0.27 & 0 & 0 & 0 & 0.06 & 0.12 & -0.20 & 0 & 0.05 & 0 & 1 & 0 & 0.04 & 0 & 0.06 & 0.24 & 0 & 0 & 0.05 \\
-0.23 & 0 & -0.12 & 0 & -0.27 & 0.03 & 0 & 0 & 0.09 & -0.05 & -0.11 & 0 & 1 & 0.14 & -0.09 & 0.12 & 0.11 & 0 & 0 & 0.16 \\
0 & 0.1 & 0 & 0.26 & 0 & 0.18 & -0.22 & 0 & 0.10 & 0.12 & 0 & 0.04 & 0.14 & 1 & 0 & -0.30 & 0 & -0.2 & 0 & 0 \\
0 & 0 & 0.03 & 0 & 0 & 0 & 0.41 & -0.11 & 0.08 & -0.14 & 0.13 & 0 & -0.09 & 0 & 1 & 0.02 & 0.03 & 0.1 & -0.14 & 0.31 \\
0 & -0.08 & 0.01 & 0.02 & -0.07 & 0 & 0 & -0.08 & 0.12 & 0.32 & 0 & 0.06 & 0.12 & -0.30 & 0.02 & 1 & -0.11 & 0 & 0.08 & 0 \\
0.12 & 0 & 0 & 0.09 & 0 & 0 & 0.05 & 0 & -0.08 & -0.06 & 0 & 0.24 & 0.11 & 0 & 0.03 & -0.11 & 1 & 0.18 & 0 & -0.15 \\
-0.21 & 0.21 & 0.11 & -0.05 & 0 & 0.08 & 0.12 & 0 & 0 & 0 & 0 & 0 & 0 & -0.2 & 0.1 & 0 & 0.18 & 1 & 0.25 & 0.24 \\
0.32 & 0 & 0 & 0 & 0.05 & 0.09 & -0.12 & 0 & 0 & -0.19 & 0.17 & 0 & 0 & 0 & -0.14 & 0.08 & 0 & 0.25 & 1 & 0 \\
0 & 0.06 & 0.23 & 0 & 0.19 & 0 & 0.12 & 0 & -0.09 & 0 & 0.11 & 0.05 & 0.16 & 0 & 0.31 & 0 & -0.15 & 0.24 & 0 & 1
\end{bmatrix}
$$

Having considered the mathematical model presented in Equations (20) to (24), we use MS-Excel solver to reach the optimal portfolio. Accordingly, the optimal project portfolio consists of implementing projects 2, 3, 4, 5, 7, 10, 11, 12, 13, 15 and 19 that covered \$296,000 out of \$300,000 total available budget. Furthermore, the expected return and risk of optimal portfolio are reached as 0.6241 and 0.0250 respectively. Figure 2 shows the optimal project portfolio and efficient frontier of the problem. It can be seen that because the decision variables are binary, the efficient frontier of PPSP is below the one drawn for the same data but considering the continuous decision variables between 0 and 1, like what is applied in PSP. This is completely logical as by adding constraints to the initial MPT's mathematical model, such as applying binary variables instead of continuous ones, the obtained efficient frontier would always fall below the initial MPT's one.

Insert Figure 2 about here

----

5. CONCLUSION

PPSP is a key decision that allows organizations reaching their strategic goals. As a project portfolio with a high attractive expected return might also expose the organization to a large loss, the combined analysis of risk and return ("risk-return optimization") should be considered in the decision model of PPSP, like what is applied in PSP. Furthermore, as there are different threats and opportunities around the projects of a portfolio, which affect their returns, risks and correlations, it is also crucial to incorporate such effects into the optimization model.

Accordingly, we proposed a new approach to such incorporation inspired by MPT and customized to PPSP through considering critical information that is not discussed in other PPSP models, i.e. particular and common threats/opportunities exist around projects.

Theoretically, this paper contributes to the literature by considering the effects of threats and opportunities around projects into the decision model through customizing MPT to PPSP. In practice, having used risk registers which are crucial documents exist in all projects, the proposed solution is asserted to be practical and effective in order to improve the reliability of portfolio selection decision in organizations.

Our paper has some limitations that need to be acknowledged. First, our study is restricted to the selection of projects to be executed. Expanding our study to the scheduling of projects in the portfolio can enrich the generalizability of the construct. Second, as our research considers the correlations among projects in the decision model, future studies can incorporate such correlations with projects' interactions, i.e. technical, resource and benefit interactions among projects derived from conducting more than one project at the same time. Third, our study applies the characteristics of projects before doing any risk mitigation, so other research can investigate the effects of candidate mitigating actions on the proposed model. Finally, we only use some sample data to demonstrate how our model should be applied. Future studies can examine the validation of the model using data from real companies.

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Table 1

The similarities between PSP and PPSP

#	Similar concepts between PSP and PPSP	Explanation
1	Return	Both assets and projects have their own expected returns.
2	Risk	There are uncertainties in the expected returns of both assets and projects.
3	Correlation	The expected returns and risks of both assets and projects are affected by external factors like the national inflation rate, which in turn can result in correlations among assets or projects. For example, a change in the inflation rate can increase the expected return of an asset or project and simultaneously lower that of another asset or project.
4	Available budget	Both investors and organizations have a limited budget in order to assign to a set of assets or projects.

Table 2

The differences between the approaches used in PSP with those should be used in PPSP

#	Parameter	PSP	PPSP
1	Required data	An appropriate amount of reliable historical data is available.	Often there is not enough historical data or the accessible historical data is not reliable enough, as the external factors around a project (i.e. the threats and opportunities faced by a project) can be unique and not possible to generalize to other projects.
2	Mean as the measure of expected return (μ_i)	Assets' expected returns are estimated by using a "backward approach". In other words, the mean of historical returns relevant to asset *i* are used to estimate its expected return.	Projects' expected return should be estimated by using a "forward approach". In other words, some forecasting techniques specific to project management should be used to estimate the expected return relevant to project *i*.
3	Standard deviation as the measure of risk (σ_i)	Assets' risks are estimated by the standard deviations of their historical returns.	Projects' risks should be estimated by using some forecasting techniques specific to project management.
4	Correlation coefficient (ρ_{ij})	The correlations among assets are calculated by using "Pearson correlation coefficient formulation" for their historical returns.	The correlations among projects should be calculated by considering the common threats and opportunities around them.
5	Benefit / Disbenefit	Only monetary benefits and disbenefits, measured in private terms, are considered in assets' expected returns.	Both monetary and non-monetary benefits and disbenefits, measured in organizational terms, should be considered in projects' expected returns.

Table 3

The required part of a project’s risk register used in estimating MPT’s parameters

ID	Threat / Opportunity	Pre-Likelihood	The damaging/contributory impacts of the threat/opportunity																				
			Benefit							Disbenefit							Cost						
			B_1		B_2		…	B_K		D_1		D_2		…	D_L		C_1		C_2		…	C_F	
			M	S	M	S	…	M	S	M	S	M	S	…	M	S	M	S	M	S	…	M	S
							…							…							…		
			ΔM	ΔS	ΔM	ΔS	…	ΔM	ΔS	ΔM	ΔS	ΔM	ΔS	…	ΔM	ΔS	ΔM	ΔS	ΔM	ΔS	…	ΔM	ΔS
							…							…							…		
							…							…							…		
							…							…							…		

Where,
“M” is the “estimated magnitude” of benefits, disbenefits or costs in the absence of any threat and opportunity,
“S” is the “estimated scheduling for the realization” of benefits, disbenefits or costs in the absence of any threat and opportunity,
“ΔM” is the “estimated percentage of changes in the magnitude” of benefits, disbenefits or costs (in two directions: increased or decreased, represented by “+” and “–” respectively) resulted from happening corresponding threat or opportunity, and
“ΔS” is the “estimated percentage of changes in the scheduling for the realization” of benefits, disbenefits or costs (in two directions: delayed or advanced, represented by “+” and “–” respectively) resulted from happening corresponding threat or opportunity.

Table 4

The required part of the risk register relevant to project 1

ID	Threat / Opportunity	Pre-Likelihood	The damaging/contributory impacts of the threat/opportunity													
			Benefit						Disbenefit		Cost					
			B_1		B_2		B_3		D_1		C_1		C_2		C_3	
			M ($)	S (Mo)*	M (I)**	S (Mo)	M (I)	S (Mo)	M (I)	S (Mo)	M ($)	S (Mo)	M ($)	S (Mo)	M ($)	S (Mo)
			12000	10	17000	11	14500	12	1000	5	5000	1	5000	5	5000	10
			ΔM	ΔS	ΔM	ΔS	ΔM	ΔS	ΔM	ΔS	ΔM	ΔS	ΔM	ΔS	ΔM	ΔS
P1.T01	Inflation rate increases	0.3	-0.07		-0.05					-0.08			+0.1		+0.16	
P1.T02	Iron import law is passed by the government	0.2			-0.08	+0.04										
P1.T03	Design changes	0.1					-0.05									

* Mo: Month
** I: Index (the unit of non-monetary benefits/disbenefits converted to dolor values by Delphi approach)

Table 5

The required part of the risk register relevant to project 2

ID	Threat / Opportunity	Pre-Likelihood	The damaging/contributory impacts of the threat/opportunity													
			Benefit				Disbenefit				Cost					
			B_1		B_2		D_1		D_2		C_1		C_2		C_3	
			M ($)	S (Mo)	M (I)	S (Mo)	M (I)	S (Mo)	M (I)	S (Mo)	M ($)	S (Mo)	M ($)	S (Mo)	M ($)	S (Mo)
			38000	9	20000	10	800	7	500	5	15000	1	2000	4	2000	7
			ΔM	ΔS	ΔM	ΔS	ΔM	ΔS	ΔM	ΔS	ΔM	ΔS	ΔM	ΔS	ΔM	ΔS
P2.T01	Inflation rate increases	0.3			-0.04									-0.07		
P2.O02	Project manager leaves	0.1	+0.07	-0.05					-0.08		-0.05					
P2.O03	Iron import law is passed by the government	0.2			+0.05		-0.05									
P2.T04	Customer requirement changes	0.2		+0.06	-0.04		+0.04									

Table 6

The return of project 1 in different potential situations

Situation (s_1)	**Threat/opportunity included in the situation**	**Probability of situation happening** ($p(s_1)$)	**Return** ($r(s_1)$)
1	P1.T01	21.6%*	41.85%
2	P1.T02	12.6%	50.67%
3	P1.T03	5.6%	55.62%
4	P1.T01 & P1.T02	5.4%	34.96%
5	P1.T01 & P1.T03	2.4%	39.49%
6	P1.T02 & P1.T03	1.4%	48.15%
7	P1.T01 & P1.T02 & P1.T03	0.6%	32.60%
8	N/A	50.4%	58.13%

* According to Equation (13): $0.3\times(1-0.2)\times(1-0.1) = 0.216$

Table 7

The returns of projects 1 and 2 in different common potential situations

Situation (c_{12})	**Threat/opportunity included in the situation**	**Probability of situation happening** ($p(c_{12})$)	**Return of project 1** ($r(c_{12})$)	**Return of project 2** ($r(c_{21})$)
1	P1.T01/P2.T01	24%*	41.85%	47.89%
2	P1.T02/P2.O03	14%	50.67%	52.87%
3	P1.T01/P2.T01 & P1.T02/P2.O03	6%	34.96%	50.53%
4	N/A	56%	58.13%	50.23%

* According to Equation (13): 0.3×(1-0.2) = 0.24

Table 8

The returns and risks relevant to 20 project proposals

Project number (i)	**Return** (μ_i)	**Risk** (σ_i)	**Required budget** (B_i)
1	51.54%	7.99%	$20,000
2	50.55%	6.58%	$32,000
3	27.52%	5.14%	$28,000
4	69.42%	8.83%	$35,000
5	49.62%	6.45%	$17,000
6	88.90%	11.57%	$23,000
7	70.06%	9.01%	$24,000
8	19.86%	3.16%	$38,000
9	33.50%	5.90%	$45,000
10	90.68%	12.80%	$33,000
11	73.37%	10.05%	$21,000
12	20.93%	3.24%	$24,000
13	94.87%	13.62%	$28,000
14	26.84%	4.80%	$40,000
15	63.76%	8.30%	$21,000
16	79.01%	10.28%	$20,000
17	40.50%	6.28%	$27,000
18	88.47%	11.52%	$35,000
19	71.15%	9.26%	$33,000
20	35.49%	6.05%	$41,000

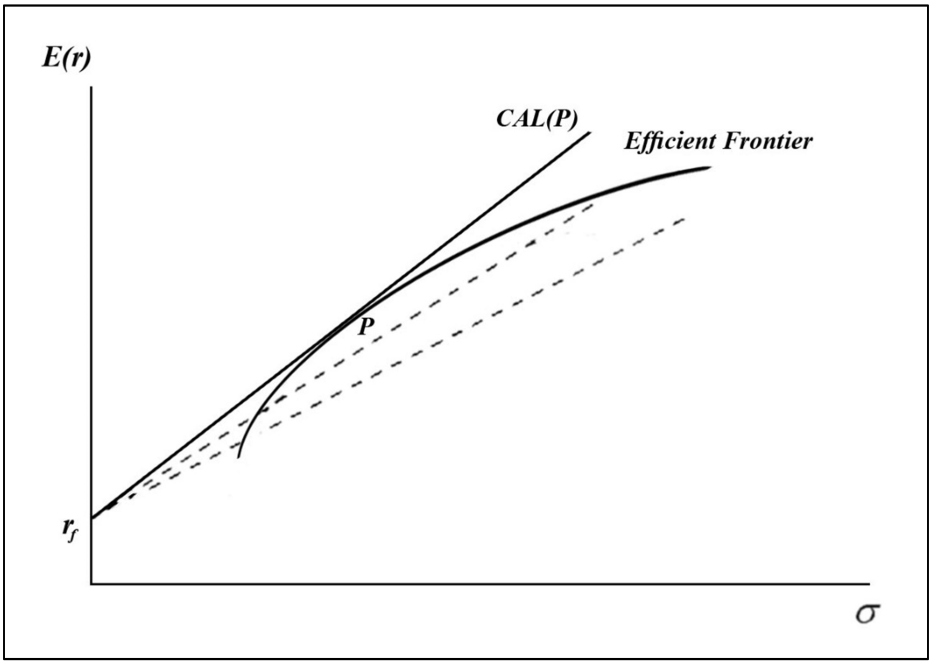

Figure 1

The efficient frontier of risky assets with the optimal capital allocation line, *CAL (P)*, and *P* as the optimal portfolio

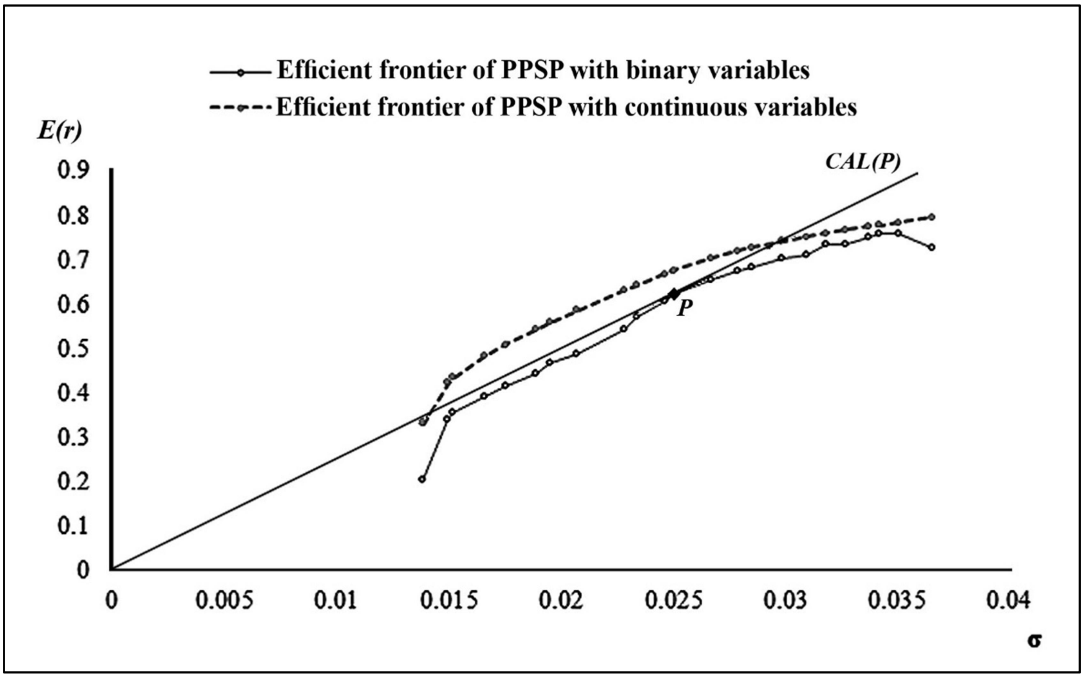

Figure 2

The efficient frontier of PPSP with binary variables compared to continuous ones as well as the optimal project portfolio (*P*)